Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**Supplementary Examination – June – 2017**

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| **Code :** | **15MA3001** | **Duration :** | **3hrs** |
| **Sub. Name :** | **ALGEBRA** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| Q. No. | Sub Div. | Questions | Course  Outcome | Marks |
| 1. | a. | State and prove fundamental theorem of arithmetic. | CO1 | 10 |
| b. | State and prove Euclid’s lemma. | CO1 | 5 |
| c. | Find the gcd (475,120) and also write gcd as a linear combination of 475 and120. | CO1 | 5 |
| (OR) | | | | |
| 2. | a. | State and prove Fermat’s theorem. | CO1 | 10 |
| b. | If the integer a has order k modulo n, then  if and only if | CO1 | 5 |
| c. | Find all primitive roots of 9. | CO1 | 5 |
| 3. | a. | State and prove Chinese Remainder Theorem. | CO1 | 10 |
|  | b. | Solve the system of congruence  ; ; | CO1 | 10 |
| (OR) | | | | |
| 4. | a. | Prove that every group is isomorphic to a subgroup of A(S) for some appropriate set S. | CO2 | 10 |
|  | b. | Prove the number of P-sylow subgroup in G for a given prime is of the form 1+kp | CO2 | 10 |
| 5. | a. | Prove that if G is a finite group and P is a prime number with Pn | O(G) and  O(G), then G has a subgroup of order Pn. | CO2 | 15 |
|  | b. | Find all the P-sylow subgroup of (Z6 , ). | CO2 | 5 |
| (OR) | | | | |
| 6. | a. | State and prove the second part of Sylow’s theorem. | CO2 | 10 |
|  | b. | If  where P is a prime, then prove that G is abelian. | CO2 | 5 |
|  | c. | Prove that a group of order 99 is not simple. | CO2 | 5 |
| 7. | a. | Prove that if R be an Euclidean ring, then any two elements a and b in R have a greastest common divisor d. Moreover d=λa+µb for some λ, µ ϵ R. | CO3 | 10 |
|  | b. | Prove that if R be a commutative ring with unit element, then prove that every maximal ideal of R is a prime ideal. | CO3 | 5 |
|  | c. | If U is an ideal of R, then prove that r(u)= {x ε R / xu=0 for all u ε U} is also an ideal. | CO3 | 5 |
| (OR) | | | | |
| 8. | a. | Let R be a Euclidean ring, then prove that every element in R is either a unit in R (or) can be written as the product of finite number of prime elements of R. | CO3 | 10 |
|  | b. | Let R be an Euclidean ring and a,b ε R. If b≠ 0 is not a unit element in R, then prove that d(a) < d(ab). | CO3 | 5 |
|  | c. | Let R be an Euclidean ring. Suppose that for a, b, c in R , a | bc but (a,b)=1, then prove that a | c. | CO3 | 5 |
|  | | **Compulsory**: |  |  |
| 9. | a. | State and prove Unique Factorization Theorem. | CO3 | 10 |
|  | b. | Prove that the set of all complex number J[i] is a Euclidean ring. | CO3 | 10 |